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IS COSMOLOGICAL ACCELERATION DRIVEN BY CLASSICAL SPACE–TIME GEOMETRY?

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The homogeneous expansion history $H(z)$ of our universe (Hubble diagram) measures only kinematic variables, it cannot fix the underlying dynamics driving the recent acceleration: cosmographic measurements of the homogeneous universe are consistent with either a static fine-tuned cosmological constant or a dynamic "dark energy" mechanism, which itself may be either material dark energy or low-curvature modifications of Einstein gravity (dark gravity). This dark energy/dark gravity degeneracy in the homogeneous expansion observations can only be resolved by observing the growth of the cosmological fluctuations. However, because the "dark energy" evolution is now quasi-static at most, any dynamical effects on the fluctuation growth function $g(z)$ will be minimal. Projected observations may potentially distinguish static from dynamic "dark energy", but distinguishing dynamic dark energy from dark gravity will require a weak lensing shear survey more ambitious than any now projected. Dark gravity is also, in principle, observable in the solar system or in isolated galaxy clusters.

The cosmological constant problem — that quantum material vacuum fluctuations apparently do not gravitate — suggests identifying gravitational "vacuum energy" with classical intrinsic space–time curvature, divorcing it from any quantum material property. This empty space–time curvature appears cosmologically and about isolated sources and can only be fine-tuned, at present. The cosmological coincidence problem — that we live when the ordinary matter density approximates the "gravitational vacuum energy" — on the other hand, is a material problem, calling for an understanding of the observers' role in cosmology. A particularly restrictive weak anthropic principle, that dark energy and dark gravity be indistinguishable, selects static "dark energy" (ΛCDM) and rejects any dynamical effects in the growth of fluctuations.

Keywords: Theoretical cosmology; phenomenological dark energy; alternative gravitational theories.

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Contents

1. Introduction: Cosmological Symmetry Versus Dynamics

The most surprising cosmological discovery is that the expansion of our universe began accelerating recently, around redshift $z \sim 0.5$. This review will stress the difference between the cosmographic and kinematic description of the expanding universe and the dynamical mechanism driving this accelerating expansion ("dark energy"). Since the homogeneous expansion $H(z)$ measures only kinematic variables, it cannot fix the underlying dynamics: cosmographic measurements of our late-accelerating universe are consistent with either a *static* cosmological constant or a *dynamic* "dark energy", which reveals either a negative pressure material within general relativity (dark energy) or modified gravity (dark gravity).[1,2] (Quotation marks are used to stress that "dark energy" and its equation of state merely summarize the expansion history, without resolving this dark energy/dark gravity degeneracy.) Indeed, the expansion history probes only one of the gravitational field equations, the original Friedmann equation within GR or a modified Friedmann equation within modified gravity.

We want to know whether "dark energy" is static or dynamic and whether it is a newly revealed material constituent within general relativity (dark energy), or a low-curvature modification of general relativity (dark gravity). Section 2 emphasizes the difference between our universe's observed *kinematics* (global homogeneity, isotropy, and spatial flatness or flat Robertson–Walker cosmology *symmetry*, RW), and the *dynamics* driving the cosmological acceleration. Section 2.4 also contains an original critique of quintessence and k-essence dark energy.

We will revert to Einstein's original definition of his cosmological constant 4Λ as a *classical* intrinsic (Ricci) space–time curvature R_{dS} of empty space–time, the ground state of gravitational theory. This identification of dark gravity with *geometry* avoids addressing the cosmological constant problem, why quantum vacuum energies apparently do not gravitate in four dimensions. This cosmological constant problem is clearly an extreme infrared gravity problem, not apparently connected with ordinary 4-dimensional quantum gravity. Brane cosmologies invoke extra compactified spatial dimensions to bridge the huge energy gap between an ordinary gravity and other fields: they solve the cosmological constant problem only by fine tuning the very small scale of these extra dimensions. Section 3 presents a controversial classical interpretation of gravitational vacuum energy and the classification of modified gravity theories by their gravitational vacuum energy.

Although "dark energy" may not be an energy at all, in order to adhere to common parlance, we denote it as "gravitational vacuum energy" $\rho_{\mathrm{vac}} := M_{\mathrm{P}}^2\Lambda = M_{\mathrm{P}}^2 R_{\mathrm{dS}}/4$, for the vacuum space–time curvature R_{dS} or the cosmological constant Λ, up to dimensional factors in the Planck mass $\mathrm{M_P}$. This intrinsic space–time curvature has some dynamical consequences, which we will discuss in Sec. 3: it distinguishes between high-curvature (ultraviolet) and low-curvature (infrared) modifications of Einstein gravity; it modifies the gravitational field surrounding an isolated source, already at a *Vainshtein radius* r_*, much smaller than the de Sitter radius r_{dS}. These examples help us to clarify the physical significance of gravitational vacuum energy and the classification of modified gravity theories by their vacuum property.

Section 4 stresses the limitations of observations of the homogeneous expansion history (Hubble diagram): the cosmography already measured in the supernova (*SN*), baryon acoustic oscillation (BAO), and CMB is simply consistent with the classical cosmological constant model (ΛCDM), but also allows a "dark energy" that is now nearly static. While Table 3 is derived from Ref. 3, its right-hand side column contains some original data.

The "dark energy equation of state" and its adiabatic sound speed c_a^2 only summarize the homogeneous expansion history $H(z)$, but cannot resolve the dark energy/dark gravity degeneracy. However, the growth function $g(z) := \delta/a$ of entropic density fluctuations $\delta := \delta\rho/\rho$ depends on the difference $c_a^2 - c_s^2$ between the adiabatic and effective sound speeds. This enables the fluctuation growth function to distinguish between static and dynamic "dark energy" and to lift the dark energy/dark matter degeneracy in homogeneous evolution. Our Chaplygin gas dark energy and the DGP dark gravity models, in Secs. 5 and 6, respectively, are original illustrations.

Weak lensing convergence observations of the fluctuation growth function potentially distinguish static from dynamic "dark energy" and dark energy from dark gravity. Because we already know that "dark energy" is either static or quasi-static, any dynamical effects in the fluctuation growth factor will, however, be small: the next decade may distinguish static from dynamic "dark energy", but will still not distinguish material dark energy from dark gravity.[4] Low-curvature modifications of Einstein gravity, may also be tested in isolated rich clusters of galaxies,[5–8] or even in precision solar or stellar system tests of anomalous orbital precession or of an increasing astronomical unit.

We stress the contrived nature of material dark energy, calling it "epicyclic", because new scalar fields are introduced *ad hoc*, only to explain dynamically the present small "gravitational vacuum energy", but do not explain the cosmic coincidence. Dark gravity modifications of classical Einstein gravity, on the other hand, which are less contrived than fine-tuned dark energy, arise naturally in braneworld cosmology, and may attempt a unification of early and late inflation. As an alternative to dark energy, we consider, in Sec. 6 the Dvali–Gabadadze–Porrati (DGP) braneworld cosmology.[8,9] This section also summarizes present laboratory and solar system tests of general relativity and dark gravity alternatives to dark energy.

Besides the original contributions listed above, this review stresses the differences between geometry (gravity) and its material sources, and between the cosmological constant and the cosmological coincidence problems, differences not generally appreciated or stressed. If the cosmological constant problem is a problem, it is a problem of empty space–time curvature: matter quantum-vacuum fluctuations apparently do not gravitate, and the small vacuum space–time curvature that is observed must be fine-tuned in any existing theory. On the other hand, the cosmological coincidence problem — why this gravitational vacuum energy and the material energy densities are just now comparable in magnitude — is a problem for intelligent material observers, which we will discuss in the concluding Sec. 7.2, our new motivation for an anthropic principle.

2. Robertson–Walker Cosmologies Describe Homogeneous Expansion

2.1. *Kinematics: Homogeneity and isotropy signify conformal flatness*

Our universe is apparently spatially homogeneous, isotropic, and spatially flat, by and large. Such Robertson–Walker cosmologies are described by the metric

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1}$$

in which the evolution of the cosmological scale $a(t) = 1/(1+z)$ with cosmic time t, is determined by gravitational field equations, which may be either Einstein's or modifications thereof. The co-moving volume element is da^3 and other kinematic observables are listed in Table 1, in which overhead dots denote derivatives with respect to cosmic time.

Table 1. Kinematic observables for any RW geometry, in terms of Hubble expansion rate $H := d\ln a/dt$ and co-moving expansion rate $\mathcal{H} := da/dt$.

Description	Definition
Hubble time	$H^{-1} := dt/dN, \quad N := \ln a$
Co-moving Hubble time	$\mathcal{H}^{-1} := 1/aH = d\eta/dN$
Expansion rate of Hubble time	$\epsilon_H := dH^{-1}/dt = -d\ln H/dN = 1 - d\ln\mathcal{H}/dN = 1 + q$
Cosmological stiffness	$\gamma(z) := 1 + w(z) := -d\ln H^2/3dN := -2\dot{H}/3H^2 := 2\epsilon_H/3$
Space expansion	$da^3/a^3 = 3dN = -d\ln(1+z)^3 = 3Hdt = 3\mathcal{H}d\eta$
Co-moving time since Big Bang	$\eta(z) := \int_0^t dt'/a(t') = \int_z^\infty dz'/H(z')$
Proper motion distance back to redshift z	$d_M(z) = c\int_0^z dz'/H(z') = c(\eta_0 - \eta(z))$
Deceleration	$\ddot{a}/a = H^2 + \dot{H}, \quad q := -\ddot{a}/aH^2 = dH^{-1}/dt - 1 = d(1/\mathcal{H})/d\eta$
Cosmological jerk	$\dddot{a}/a = H^3 + 3H\dot{H} + \ddot{H}, \quad j := \dddot{a}/aH^3 = 1 + 3\dot{H}/H^2 + \ddot{H}/H^3$
Spacetime (Ricci) curvature	$R := 6(k/a^2 + \dot{H} + 2H^2) = 6(k/a^2 + (\dot{a}/a)^2 + \ddot{a}/a) = 6(k/a^2 + H^2(1-q))$

The RW metric is conformally flat, meaning that it can be re-written

$$ds^2 = a^2[-d\eta^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{2}$$

in terms of a conformal or co-moving time defined by $a(t)d\eta := dt$. This conformal flatness makes light propagate in every co-moving frame as in Minkowski space, so that the observable quantities in Table 1 are all either geometric or kinematic observables. Assuming the equation of state $w \geq -1$, the weak energy condition $\rho + P \geq 0$ on matter excluding phantom energy in GR, there is no cosmological big rip and the inflationary de Sitter universe $a(t) \propto t^{2/3(1+w)}$ is an attractor for expanding RW universes. The Hubble time then increases continuously: $\epsilon_H := dH^{-1}/dt = d\mathcal{H}^{-1}/d\eta + 1 > 0$. But, if the co-moving Hubble expansion rate reaches a minimum, w falls below $-1/3$, the deceleration $q := (1+3w)/2 = d(1/\mathcal{H})/d\eta$ becomes an acceleration, and the co-moving Hubble expansion rate starts increasing with co-moving time $d\mathcal{H}/d\eta > 0$. This inflationary change from deceleration to acceleration transpired in the very early universe and again recently at $z \sim 1/2$ (Fig. 1). While early inflation proceeded until the parameter $\epsilon_H \ll 1$, the current inflation has started only recently, so that, although $\epsilon_H < 1$, it does not yet deserve the appellation "slow-roll parameter".

2.2. *Dynamics: General relativity or modified gravity?*

In any metric theory of gravitation, the space–time curvature (Riemann tensor) consists of a trace-free (Weyl) tensor part and a remainder (Ricci) tensor. These are

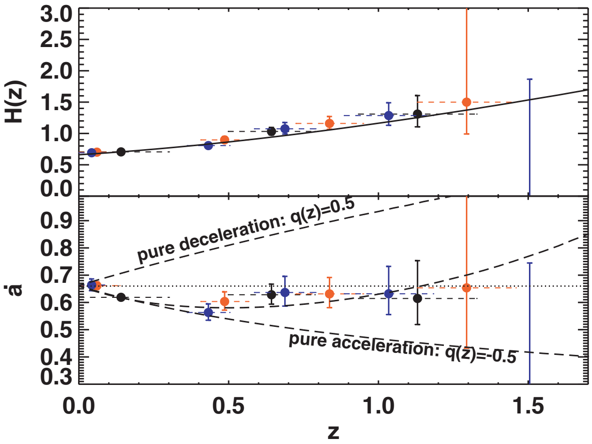

Fig. 1. Upper panel: Hubble expansion rate, in units of 100 km/sec/Mpc, derived from the supernova gold sample, compared with the concordance model with $\Omega_{m0} = 0.29$, $\Omega_\Lambda = 0.71$, for which acceleration starts at $z \sim 0.7$. Lower panel: conformal Hubble expansion rate $\mathcal{H} = \dot{a} = H(z)/(1+z)$, compared to the constant deceleration model $q(z) = 0.5$ (top dashed curve), coasting model $q(z) = 0$ (middle dotted curve), constant acceleration model $q(z) = -0.5$ (bottom dashed curve). The gold data favors the recently accelerating model $q(z) \approx -0.6 + 1.2z$ (middle dashed curve), with an acceleration starting at $z \sim 1/2$ (from Riess *et al.*,[10] Fig. 7).

determined by the matter stress-energy distribution nonlocally and locally, respectively. The only gravitational degrees of freedom are those locally connected to matter through the Ricci tensor, which is subject to four differential Bianchi identities $\nabla^\mu R_{\mu\nu} \equiv \nabla_\nu R$, following from general co-variance and reducing the number of degrees of freedom to six in $R_{\mu\nu}$ and $g_{\mu\nu}$. Besides the metric $g_{\mu\nu}$, the Einstein tensor $G_{\mu\nu} := R_{\mu\nu} - g_{\mu\nu}R/2$ is the only co-variantly conserved second-rank tensor. This suggests making the matter tensor $T_{\mu\nu}$ proportional to $G_{\mu\nu} + \Lambda g_{\mu\nu}$ or some function thereof.

Now, the Hubble expansion rate $H(t)$ derives from the gravitational field equations:

- *In Einstein's original general relativity*, the field equations are

$$G_{\mu\nu} = \varkappa^2 T_{\mu\nu}, \quad \varkappa^2 := 8\pi G_{\rm N} := M_{\rm P}^{-2}, \tag{3}$$

where $G_{\rm N}$ is Newton's constant, and $M_{\rm P}$ is the reduced Planck mass. In RW cosmology, there is only one independent field equation, leading to the Friedmann equation

$$H(a)^2 := \frac{\dot{a}^2}{a^2} = \frac{\varkappa^2 \rho_m}{3} - \frac{k}{a^2}, \tag{4}$$

where ρ_m is the material energy density, which vanishes in empty space. The vacuum space–time curvature $R_{\rm dS} = 0$.

- *In Einstein–Lemaître general relativity*, the field equations become
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \varkappa^2 T_{\mu\nu}, \tag{5}$$
and the Friedmann–Robertson–Walker (FRW) equation and the vacuum space–time curvature become
$$H(a)^2 := \frac{\dot{a}^2}{a^2} = \frac{(\varkappa^2\rho_m + \Lambda)}{3} - \frac{k}{a^2}, \quad R_{dS} = 4\Lambda. \tag{6}$$
For both the original Einstein and the later Einstein–Lemaître field equations, the linearity in $G_{\mu\nu} + \Lambda g_{\mu\nu}$ guarantees local conservation of matter stress-energy $T_{\mu\nu}$ and the weak equivalence principle. $H(t)$ is the only cosmological degree of freedom, and only the tensor components of the metric $g_{\mu\nu}$ are propagating.
- *In alternative gravitational theories*, $T_{\mu\nu}$ is no longer simply proportional to $G_{\mu\nu} + \Lambda g_{\mu\nu}$ and $\dot{H}$ and $\ddot{H}$ become additional scalar degrees of freedom. Because these scalars couple nonminimally to matter, the weak equivalence principle and Newtonian inverse square gravity may no longer be satisfied at short distances. H^2 is nonlinear in the matter density ρ_m and the modified Friedmann equation now incorporates only one of the independent field equations.

The RW symmetry is broken at small cosmological scales, where inhomogeneities occur. These inhomogeneities or fluctuations break translational invariance, leading to Goldstone mode sound waves and the growth of structure, starting with the formation of protogalaxies at about redshift $z \sim 10$. This symmetry-breaking at low temperatures is reminiscent of symmetry-breaking in condensed matter and particle physics, except that cosmological structures are gravitationally unstable and will ultimately collapse or decay away in an expanding universe.

2.3. *Hydrodynamic description of "dark energy" and "equation of state"*

The expansion history does not determine the dynamics. Although flat RW kinematics does not assume general relativity, by defining
$$\rho_{\mathrm{DE}} := 3M_{\mathrm{P}}^2 H^2 - \rho_m, \quad \gamma_{\mathrm{DE}}(z) := -d\ln\left(\frac{\rho_{\mathrm{DE}}}{3dN}\right), \tag{7}$$
the homogeneous expansion history may be described by a two-component perfect fluid:

composite mass density: $\rho := 3M_{\mathrm{P}}^2 H^2 := \rho_m + \rho_{\mathrm{DE}}$
composite pressure: $P := -M_{\mathrm{P}}^2 c^2(3H^2 + 2\dot{H}) := P_m + P_{\mathrm{DE}}$
composite enthalpy density: $\rho + P/c^2 := -2M_{\mathrm{P}}^2\dot{H} = -d\rho/3dN$
composite fluid stiffness: $\gamma(z) := -d\ln\rho/3dN := 1 + w = -2\dot{H}/3H^2 = \gamma_m\Omega_m + \gamma_{\mathrm{DE}}(1 - \Omega_m)$
composite "equation of state": $w(z) := -(3H^2 + 2\dot{H})/3H^2 = d\ln(H^2/(1 + z)^3)/3dN = w_m\Omega_m + w_{\mathrm{DE}}(1 - \Omega_m)$
adiabatic sound speed: $c_a^2 := dP/d\rho = \dot{P}/\dot{\rho} := -(1 + \ddot{H}/3H\dot{H})$.

Integrating $\gamma_{\mathrm{DE}}(z)$, $\rho_{\mathrm{DE}}(z) = \rho_{\mathrm{DE0}}(1+z)^{3\overline{\gamma_{\mathrm{DE}}(z)}}$, where $\overline{\gamma_{\mathrm{DE}}}(z) := (1/N) \int_0^N \gamma_{\mathrm{DE}}(N')\,dN'$ is the past-averaged value of the "dark energy stiffness", so that the observed Hubble expansion

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})(1+z)^{3\overline{\gamma_{\mathrm{DE}}(z)}}. \tag{8}$$

The departure from homologous expansion, or the curvature in the Hubble expansion rate $\mathcal{H}$ (Fig. 1), marks the appearance of "dark energy". The present acceleration requires $\gamma_{\mathrm{DE0}} < 2/3$, so that the "dark energy" is diluting slower than the matter density.

Apparently, after a high-curvature early inflationary phase, our universe expanded monotonically through radiation-dominated and matter-dominated phases, towards a different low-curvature late inflationary phase. During each of the four barotropic phases in Table 2, the equation of state, adiabatic sound speed, acceleration, and jerk were constant and the universe expanded homologously towards smaller space–time curvature. During phase transitions that mix these perfect fluids or introducing cosmological scalar fields, the equation of state changes, making the composite fluid imperfect, generating entropy, and breaking the homologous expansion. Assuming that no phantoms intervene to make $w < -1$, the universe will asymptote monotonically towards a future de Sitter phase of small, but finite, space–time curvature R_{dS}.

The bottom portion of Fig. 1, from Riess *et al.*,[10] shows the co-moving Hubble expansion rate $\mathcal{H} = 0.68(1+z)^q$ for three hypothetical phases with constant deceleration $q(z) = 0.5$ (upper dashed curve), constant acceleration $q(z) = -0.5$ (lower dashed curve), and coasting $q(z) = 0$ (central dotted line). The supernova data is fitted by the central dashed curve $q(z) = -0.6 + 1.2z$, changing from deceleration to acceleration around $z = 0.46$ when the co-moving expansion rate reached a broad minimum $\mathcal{H}(z) \sim 0.6$.

2.4. *"Dark energy" is now static or very nearly static*

"Dark energy" and its evolution, $\rho_{\mathrm{DE}}(z)$ and $w_{\mathrm{DE}}(z)$, no more than summarize the accelerated expansion history of our universe. This "dark energy" is either a newly revealed material constituent within GR or a modification of GR.

Dark Energy: In general relativity, the total matter stress-energy is co-variantly conserved and the FRW equation (6) is the only independent field equation. For a given form of kinetic energy with $X := \partial_\mu\phi\partial^\mu\phi/2 > 0$, $\dot{\phi}$ lets the field $\phi(t)$ substitute for the time. If the scalar field is *canonical* (kinetic energy density $\partial_\mu\phi\partial^\mu\phi/2 := X$), then $\dot{\phi}^2 = (1+w_{\mathrm{DE}})\rho_{\mathrm{DE}}$, so that the expansion history determines the potential energy density $V(\phi) = (1-w_{\mathrm{DE}})\rho_{\mathrm{DE}}/2 = (1-w_{\mathrm{DE}})(3M_{\mathrm{P}}^2H^2 - \rho_m)/2$. If the scalar field is *noncanonical* (kinetic energy density nonlinear in X), w_{DE} determines a different potential.

Table 2. Five barotropic phases of our flat universe expanding homologously as $a \sim t^{1/\epsilon_H}$ or $a \sim \exp Ht$, with constant equation of state $w = \gamma - 1 = 2\epsilon_H/3 - 1$.

γ	w	$a(t) \sim t^{1/\epsilon_H}$	$H(t) \sim 1/\epsilon_H t$	$q(t) = (1+3w)/2$	$j(t) = 1 + 9w(1+w)/2$	$R(t) = 6H^2(1-q)$	Model flat universe
4/3	1/3	$t^{1/2}$	$1/2t$	1	3	0	radiation-dominated
1	0	$t^{2/3}$	$2/3t$	1/2	1	$3H^2$	matter-dominated (*E*-dS)
2/3	−1/3	t	$1/t$	0	0	$6H^2$	coasting
1/3	−2/3	t^2	$2/t$	−1/2	0	$9H^2$	accelerating
0	−1	$\exp H_{\mathrm{dS}} t$	$H_{\mathrm{dS}} = \mathrm{const}$	−1	1	$12H_{\mathrm{dS}}^2 = 4\Lambda$	inflationary (de Sitter)

Dark Gravity: If no such dark energy exists, then $w_{\mathrm{DE}}(z) := w(z)/(1-\Omega_m(z))$ *defines* the dark gravity modification to the Friedmann equation. If dark gravity is dynamic, then the modified Friedmann equation is only one of the field equations.

If dynamical, both dark gravity and dark energy lead to scalar fields. These new scalar fields are introduced as negative pressure matter in dark energy, or are realized as new gravitational scalar degrees of freedom in dark gravity, i.e. the Friedmann equation (4) is modified either on the left-hand side or on the right-hand side.

Dynamical dark energy was originally invoked to make a material vacuum energy decay down to the present small value $\rho_{\mathrm{vac}0} \sim 2M_{\mathrm{P}}^2 H_0^2 \lll M_{\mathrm{P}}^4$ or small expansion rate $H_0^2 \lll M_{\mathrm{P}}^2$. This and some of its alternatives are reviewed in Refs. 11 and 12, and ten model fits to the expansion history, with and without spatial curvature and cosmological constant, are tabulated by Ref. 13. If dark energy exists, it is usually attributed to an additional ultralight scalar field ϕ with Lagrangian $\mathcal{L}(X,\phi)$, where the canonical kinetic energy density $X := \partial_\mu\phi\partial^\mu\phi/2$, the pressure $P(X,\phi) = \mathcal{L}$, the energy density $\rho(\phi) := 2X\mathcal{L}_{,X} - \mathcal{L}$, and the "equation of state" $w := P/(2XP_{,X} - P)$. Provided $\mathcal{L}_{,X} \geq 0$, the null energy condition $w \geq -1$ bounds the energy density from below, so that the system is stable. Perturbations propagate with the effective sound group velocity $c_s^2 := P_{,X}/\rho_{,X} = (\partial P/\partial\rho)_{\phi=\mathrm{const}} \geq 0$.

Depending on whether the kinetic energy is linear or nonlinear in X, the Lagrangian $\mathcal{L}$ is canonical or noncanonical.

Quintessence is canonical, with a slow-rolling scalar potential energy density that can be tuned to track the radiation/matter until it dominates now, with an equation of state now decreasing: $dw_{\mathrm{DE}}/dz > 0$ and $c_s^2 = 1$.

***K*-essence** is noncanonical, with a kinetic energy density chosen to track only the radiation energy density so that, after radiation/matter equality, k-essence can start dominating over ordinary matter. w_{DE} dropped sharply near matter/radiation equality and has been increasing thereafter: $dw_{\mathrm{DE}}/dz < 0$. Although k-essence evolves to superluminal sound group velocities, $c_s^2 \geq 1$, this does not lead to signal velocities exceeding light velocities in vacuum or causal paradoxes.[14]

Both kinds of dark energy ultimately require fine tuning in different ways: quintessence, in order to stop tracking before now; k-essence, in order to initiate the transition towards dominance in the matter-dominated epoch. Because these scalar fields are non-renormalizable and fundamentally unnatural, both need to be interpreted as *ad hoc* low-energy effective field theories.

According to WMAP3,[15] the present Hubble expansion rate $H_0 = \mathcal{H}_0 = 73 \pm 3\,\mathrm{km/sec/Mpc}$, Hubble time $H_0^{-1} = \mathcal{H}_0^{-1} = 13.4 \pm 0.5$ Gyr $= 4.11 \pm 0.17$ Gpc, so that the cosmic acceleration has only increased to $-q_0 \approx 0.52$, the "slow-roll" parameter has only decreased to $\epsilon_{\mathrm{H}0} = 0.48$, the overall "equation of state" is now

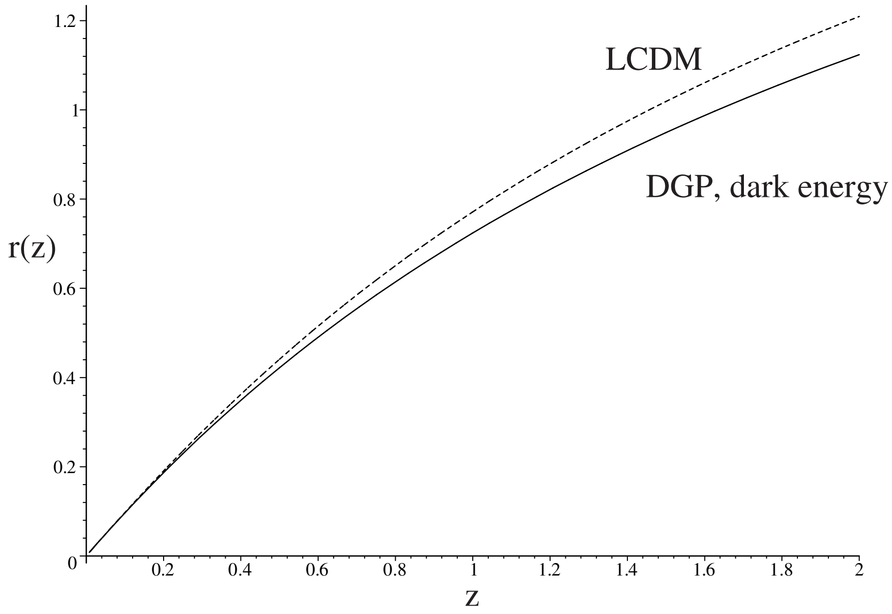

Fig. 2. The co-moving or proper motion distance $r(z) \equiv d_M(z) := c\int_0^z dz'/H(z')$ back to the redshift z, shows curvature at high redshift depending on the cosmological model. Distances obtained from the dynamic DGP dark gravity model or its equivalent dark energy mimic are somewhat less than those obtained from the static LCDM model because the DGP gravity was somewhat stronger in the past.[17]

$w_0 \approx -0.74$. For constant w, the SNLS supernova data then implies $\Omega_{m0} = 0.234 \pm 0.035$ and the "dark energy equation of state" $w_{\mathrm{DE}0} = -0.926^{+0.051}_{-0.075}$ (Ref. 15). This limit as to how dynamic the dark energy equation of state may be is consistent with ΛCDM, and will improve when more weak lensing measurements of galaxy halo masses and cluster abundances lead to an improved constraint on the matter spectrum amplitude σ_8, the rms mass dispersion on a sphere of radius $8\,h^{-1}$Mpc.[16]

The lower curve in Fig. 2, from Koyama and Maartens,[17] shows the dark gravity/dark energy degeneracy in DGP co-moving distance $d_{\mathrm{M}}(z)$ derived from the DGP expansion history. In Sec. 5, we will see how this dark energy/dark gravity degeneracy in homogeneous evolution may be resolved by prospective observations of the growth of inhomogeneities.

3. Properties of the Gravitational Vacuum

3.1. *Classical cosmological constant model* Λ*CDM*

Einstein added his cosmological constant $\Lambda g_{\mu\nu}$ on the left (geometric) side of his original field equations by changing them to the Einstein–Lemaître form as in Eq. (5) and changing the original Einstein Lagrangian R into the Einstein–Lemaître

Lagrangian $R - 2\Lambda$. (Equivalently, the original Einstein Lagrangian R can be varied holding $\sqrt{g} = -1$. In this unimodular gravity approach,[18,19] Λ does not appear in the Lagrangian, but as an undetermined c-number Lagrange multiplier. This geometric approach stresses the classical nature of the cosmological constant and does not attempt to compute the cosmological constant.) In fact, one might argue that space–time curvature is the only way to give content to the notion of "empty space".

The alternative dark matter interpretation subtracts $\Lambda g_{\mu\nu}$ from the right-hand side of Eq. (5) and interprets the cosmological constant physically as a constant-density fluid with $\rho_{\text{vac}} := M_{\text{P}}^2\Lambda = (2.39\ \text{meV})^4$. These geometric/physical interpretations of the cosmological constant already exhibit the dark gravity/dark energy degeneracy in the expansion history.

Allowing for possible space curvature, the FRW equation (6) contains two parameters, namely Λ and the present energy density ρ_{m0}, which is now almost completely that of nonrelativistic matter. In units of the present critical density, $\rho_{\text{cr0}} := 3M_{\text{P}}^2H_0^2$,

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{K0}, \quad \Omega_{m0} + \Omega_{\Lambda 0} + \Omega_{K0} \equiv 1, \tag{9}$$

where Ω_{m0} and $\Omega_{\Lambda 0}$ are the present matter and vacuum fractions and $w_{\text{DE}} = -1$.

The observed electromagnetic Casimir effect and CMB anisotropies show only that vacuum quantum *fluctuations* gravitate, but are not evidence that material vacuum energies themselves gravitate.[20] Indeed, the tiny observed value for the cosmological constant suggests the *inequivalence* of the gravitational and material vacua! (This interpretation is opposed to that of quantum gravity, which seeks to unify gravity and matter, and interprets space–time curvature materially as "gravitational vacuum energy".)

3.2. *Classical geometric interpretation of vacuum space–time curvature*

Robertson–Walker symmetry has some dynamical consequences, which help us to elucidate the physical consequences of vacuum space–time curvature and the connection between geometry and material sources.

In general relativity, the Bianchi identities assure co-variant conservation of the material stress-energy. This will no longer be generally true for dark gravity, where the material stress-energy tensor $T_{\mu\nu}$ is no longer simply proportional to $G_{\mu\nu} + \Lambda g_{\mu\nu}$ and is not generally locally conserved. Nevertheless, in empty space, the Ricci scalar and the Hubble expansion rate will still asymptote to the small constant values $R_{\text{dS}} = 4\Lambda$. We will interpret dark gravity classically and geometrically. This interpretation, which is at odds with that of quantum gravity, avoids the cosmological constant problem by simply fine tuning the geometry to $\Lambda := 3H_{\text{dS}}^2 = 2.19H_0^2 \approx 1.29 \times 10^{-56}\,\text{cm}^{-2}$ or de Sitter radius $r_{\text{dS}} \sim 5.2\,\text{Gpc}$.

Two empty RW cosmologies emphasize the important distinction between space–time and spatial curvature:

Milne model: It has negative spatial curvature $k = -1$, but vanishing space–time curvature. $H(a)^2 = 1/a^2$ so that the scale is expanding uniformly $\mathcal{H} = \dot{a} = 1$ and Hubble's original linear relationship between redshift and distance remains exact at all redshifts.

Spatially flat de Sitter model: $a(t) = \exp H_{\mathrm{dS}} t$ has constant space–time curvature $R = 4\Lambda$ and de Sitter horizon $r_{\mathrm{dS}} := \sqrt{3/\Lambda}$. Within this horizon, the metric can be put into the static form:

$$ds^2 = -\left(\frac{1 - r^2}{r_{\mathrm{dS}}^2}\right) dt^2 + \frac{dr^2}{1 - r^2/r_{\mathrm{dS}}^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad r < r_{\mathrm{dS}}. \quad (10)$$

In the very far future, when $t \sim 800\,\mathrm{Gyr}$, although our local group will still contain long-lived stars, it will become causally disconnected from the expanding universe outside this horizon. We will be able to still observe proper motions within our local group, but the global Hubble expansion and "dark energy" will then become unobservable. This makes "dark energy" unobservable in both the early universe, when $\rho \gg \rho_{\mathrm{vac}}$, and in the very far future, when $\rho \lll \rho_{\mathrm{vac}}$.[21,22]

The Riemann curvature tensor can always be decomposed into a traceless part (Weyl or conformal tensor) plus a remaining part (Ricci tensor), which is determined by the local matter distribution. In RW cosmologies, the (4-dimensional) Weyl tensor vanishes and Ricci scalar depends on the acceleration imparted to matter by the field equations. We will now consider the gravitational vacuum in these two limits: RW cosmology (vanishing Weyl tensor) and the vacuum about a spherically symmetric source (vanishing Ricci tensor).

3.2.1. *Robertson–Walker cosmologies classified by vacuum space–time curvature*

Free of dark energy, our accelerating universe is now dominated by pressure-free matter so that dark gravity must modify the Friedmann equation at cosmological distances, while still preserving Einstein gravity and the equivalence principle locally. The alternative high-curvature modifications require sub-millimeter corrections to Newton's inverse-square gravity. In this way, low- and high-curvature modifications of general relativity are distinguished by the presence or absence of gravitational vacuum energy. While the gravitational vacuum may be intrinsically classical in origin, the high-curvature modifications may be classical[23] or quantum gravity[24,25] in origin.

3.2.2. *Vacuum about an isolated spherically symmetric source: Vainshtein radius*

The gravitational field about any isolated spherically symmetric source of mass M or Schwarzschild radius $r_S := 2G_N M$ is different in GR and in braneworld cosmology.

- In general relativity, the vacuum field equations are the vanishing of the Einstein tensor $G_{\mu\nu} := R_{\mu\nu} - Rg_{\mu\nu}/2 = 0$, and the unique spherically symmetric vacuum metric is the *Schwarzschild–de Sitter metric*:
$$g_{tt} = g_{rr}^{-1} = \frac{1 - r_S}{r} + \frac{\Lambda r^2}{3}. \tag{11}$$
A vanishing vacuum energy would imply Birkhoff's theorem, a generalization of Newton's Iron Sphere theorem: for any thin spherical shell, the gravitational potential vanishes inside, and decreases outside as $1/r$. (From this clear geometric theorem, Milne and McCrae[26,27] once derived Friedmann equation $k/a^2 + H^2 = \varkappa^2\rho/3$ for a pressure-free universe, assuming only Newtonian gravity. In such a dust universe, Newtonian cosmology would then have implied Friedmann equation! (Of course, in Newtonian cosmology, space would always be flat, so that the spatial curvature k and scale factor a would lack the geometrical interpretation GR conveys.) Because vacuum energy is now known to exist, Birkhoff's theorem and the Milne–McCrae derivation is today only an historical curiosity.)
- Birkhoff's theorem does not apply in braneworld cosmologies. In the Dvali–Gabadadze–Porrati Model, discussed in Appendix C, the field equations are the vanishing of $G_{\mu\nu} + E_{\mu\nu} = 0$, where $E_{\mu\nu}$ is the projection of the 5-dimensional Weyl tensor onto the 4-dimensional brane, whose dynamical importance will be apparent in Fig. 3. The DGP vacuum metric is:
$$g_{tt} = \frac{1 - r_S}{2r} + \sqrt{\frac{r_S r}{2r_c^2}}, \quad g_{rr}^{-1} = \frac{1 + r_S}{2r} - \sqrt{\frac{r_S r}{8r_c^2}}, \quad r \lesssim r_\star := (r_S r_c^2)^{1/3}, \tag{12}$$
where r_c is a new cosmological scale. Because this metric differs from the Schwarzschild metric, even when $r_c \to \infty$, there are no DGP iron sphere or Birkhoff's theorems. This emphasizes the characteristic geometric nature of these theorems in general relativity and the distinctive role of the 5-dimensional Weyl tensor in brane cosmology.

In both cases, the vacuum energy modifies the Schwarzschild metric at distances beyond the *Vainshtein radius* $r_* := (r_S r_{\mathrm{dS}}^2)^{1/3}$, where the de Sitter radius r_{dS} is H_0^{-1} or $r_c = \beta H_0^{-1}$, for ΛCDM and for DGP, respectively. This geometric mean between r_S and r_c^2 will also be where fluctuations start increasing according to Friedmann–Lemaître or linearized DGP,[28] instead of according to Einstein gravity. These Vainshtein scale modifications may potentially be observable in measurements about isolated Sun-like stars ($r_S \sim 3\,\mathrm{km}$, $r_\star \sim 280\,\mathrm{pc}$) or spherical galaxy clusters ($r_S \sim 100\,\mathrm{pc}$, $r_\star \sim 28\,\mathrm{Mpc}$).[6] Around distant galaxies or rich clusters, the 5–27% reduction in relativistic bending of light is already nearly observable.[29] Higher-order effects may also someday be tested in ultraprecise solar system measurements of anomalous precessions of planetary or lunar orbits[5,30,31] or of a secular increase in the astronomical unit.[7]

To summarize without dark energy, the accelerating universe requires vacuum space–time curvature, called gravitational vacuum energy. Besides its cosmological implications, gravitational vacuum energy implies deformation of the Schwarzschild metric about any isolated source at the Vainshtein radius r_*, which is significantly smaller than the de Sitter radius.

4. Homogeneous Expansion Measures Only Kinematic Variables

While realizing that the homogeneous expansion cannot resolve the dark energy/dark gravity degeneracy, we finally review both static and dynamic fits to the observed expansion history.

4.1. *Cosmography: Distances to supernovae, luminous red galaxies, and last scattering surface*

For small radial distances and small galaxy recessional velocities, Hubble's law $v = cz = H_0 d$ is a kinematic consequence of RW symmetry, illustrated by the linear region $d = cz/H_0$ in Fig. 2. But, in curved space–time, different global distances are defined only by physical observables: the co-moving distance back to a source defines the proper motion distance $d_{\rm M}(z) := c(\eta_0 - \eta(z))$; the observed flux $\mathcal{F} := \mathcal{L}/4\pi d_{\rm L}^2$ from standard candles defines the luminosity distance $d_{\rm L}(z) := \sqrt{4\pi\mathcal{F}/\mathcal{L}} = (1+z)d_{\rm M}$; the angular size θ_A of a standard ruler r_s defines the angular diameter distance $d_{\rm A}(z) := r_s(z)/\theta_{\rm A} = d_{\rm M}/(1+z)$. These different cosmological distances are observed as described below.

CMB: The angular diameter distance of the first acoustic CMB peak at the last scattering surface measures the co-moving size subtended at an angular scale $\theta_{\rm A}$. The measured CMB shift parameter $S := \sqrt{\Omega_{m0}}H_0 d_{\rm M}(z_r) = 1.70 \pm 0.03$ then determines the distance to the last scattering surface at $z_{\rm ls} = 1089$ and a standard ruler, the co-moving sound horizon $r_{\rm s} = 147.8^{+2.6}_{-2.7}$ Mpc.[15,32]

BAO: This provides a standard ruler for measuring the line-of-sight distances $H(z)r_{\rm s}$ to luminous red galaxies (LRG) and their angular size $d_{\rm A}(z)/r_{\rm s}$. From the $z_1/z_{\rm ls}$ distance ratio, $R_{0.35} = 0.0979 \pm 0.0036$ and the measured combination $A := \sqrt{\Omega_{m0}}H_0[d_{\rm M}^2(z_1)/z_1^2 H(z_1)]^{1/3} = 0.469 \pm 0.017$, Fairbairn and Goobar[33] and Eisenstein *et al.*[34] obtained the proper motion distance $d_{\rm M}(z_1)$ to LRGs, typically at redshift $z_1 = 0.35$.

SN: The luminosity distances of calibrated supernovae Ia are derived directly from their observed fluxes.[10,35–37] The quality data[10,36,37] is now dominated by systematic errors due to the nearby velocity structures and dust.

WL: Weak gravitational lensing of the light from galaxy clusters, from the X-ray emission from hot gravitationally confined electrons, and from the upscattering of CMB relict photons by these hot electrons (Sunyaev–Zelovich effect) measures the proper motion distances of these sources and the fluctuation growth factors $g(z)$ at these distances. Weak lensing observations are independent of the baryonic composition of the lenses and enjoy a statistical potential far greater than BAO or SN.

GC: Galaxy clustering also measures both $d_{\rm M}(z)^2/H(z)$ and $g(z)$, but is subject to large systematic errors, deriving from their baryonic composition and foreground noise.

The proper distance $c(\eta_0 - \eta(z)) := d_{\rm M}(z) = (1+z)d_{\rm A}(z) = d_{\rm L}(z)/(1+z)$, must be differentiated with respect to redshift, to obtain Hubble times $H(z)^{-1} = d\eta/dz$, $\mathcal{H}^{-1} = d\eta/d\ln 1+z$. Obtaining the composite equation of state $w(z)$ and the dark energy equation of state $w_{\rm DE}(z) = w(z)/(1-\Omega_m(z))$, requires a second differentiation of the observed distances. This requires smoothing and binning of the data,[38] smears out information on the "equation of state",[39] and justifies no more than two-parameter models[40,41] in fitting current and currently underway observations. The usual Chevalier–Polarski–Linder parameterization[42,43] of the dark energy equation of state

$$w_{\rm DE}(z) = w_0 + w_a(1-a), \quad \overline{w_{\rm DE}(z)} = w_0 + w_a(1-a)\ln a \tag{13}$$

assumes that "dark energy" increases smoothly, monotonically, and mostly at low redshifts, a prior assumption that will be tested only after more observations at higher redshifts become available. More generally, because observations constrain the directly observable $H(z)$ and the "dark energy" density $\rho_{\rm DE}$ better than its derivative, it might be better to parameterize the past average $\overline{w_{\rm DE}}(z)$, rather than $w_{\rm DE}(z)$.[44]

4.2. *Dark energy/dark gravity degeneracy persists in dynamical cosmological models*

The cosmological constant model can be made dynamic by introducing additional parameters relaxing the condition $w = -1$. In such models, the dark energy/dark gravity degeneracy persists, but the gravitational vacuum energy decays down to its present observed value. No model explains the cosmic coincidence, namely why we are observing the universe now when the present matter density $\rho_{m0} \sim \rho_{\rm vac}/3$.

Table 3, derived from Davis *et al.*[3] with some additions, tabulates 12 "dark energy" fits to the SN + CMB + BAO homogeneous evolution data, ordered according to the Schwarz Bayes information criterion (BIC), an approximation to the marginal likelihood of improving the fit by adding more parameters, which measures the strength of each model in giving the best fit with the fewest parameters. Although some of these fits derive from interesting dark gravity models, their homogeneous evolution can always be mimicked by equivalent dark energy models.

Table 3. Classical cosmological constant and dynamical models for the homogeneous evolution. The goodness of fit (GoF) approximates the probability of finding a worse fit to the data. The Bayes information criteria (BIC) prefer the one-parameter flat cosmological constant model. The ΔBIC values for the other two- and three-parameter models are then measured with respect to this flat cosmological constant model. The table is derived from Davis *et al.*,[3] Table 2, ordered by increasing the complexity of ΔBIC, but with some original additions to the last column.

Model	χ^2/dof	GoF(%)	ΔBIC	Parameters fitted
Flat cosmologic constant (6, $k = 0$)	194.5/192	43.7	0	$\Omega_{m0} = 0.27 \pm 0.04$
Flat generalized Chaplygin gas (14)	193.9/191	42.7	5	$A = 0.73 \pm 0.04,\ \alpha = 0.05 \pm 0.10$
Cosmological constant (12)	194.3/191	42.0	5	$\Omega_{m0} = 0.29 \pm 0.04,\ \Omega_{K0} = -0.016^{+0.030}_{-0.029}$
Flat constant EOS w_{DE}	194.5/191	41.7	5	$\Omega_{m0} = 0.27 \pm 0.04,\ w_{\mathrm{DE}} = -1.01 \pm 0.15$
Flat variable $w_{\mathrm{DE}}(z)$ (11)	193.8/190	41.0	10	$\Omega_{m0} = 0.27 \pm 0.04,\ w_0 = -1.0 \pm 0.4,\ w_a = -0.4 \pm 1.8$
Spatially curved constant w_{DE}	193.9/190	40.8	10	$\Omega_{m0},\ w_{\mathrm{DE}},\ \Omega_\Lambda$
Generalized Chaplygin gas (15)	193.9/190	40.7	10	$A,\ \alpha,\ \Omega_{K0}$
Cardassian polytropic (16)	194.1/190	40.4	10	$\Omega_{m0},\ q,\ n$
Flat Dvali–Gabadadze–Porrati (18, $k = 0$)	210.1/192	17.6	14	$\Omega_{m0} \approx 0.27$
Dvali–Gabadadze–Porrati (18)	207.4/191	19.8	18	$\Omega_{m0} = 0.27 \pm 0.05,\ \Omega_{K0} = 0.13 \pm 0.02$
Ordinary Chaplygin gas (15, $\alpha = 1$)	220.4/191	7.1	30	$A,\ \Omega_{K0}$
Flat ordinary Chaplygin gas (14, $\alpha = 1$)	301.0/192	0	30	A

The first 8 of these 12 models fit the combined data almost equally well. But, ordered by BIC, 12 models fall into 4 categories of increasing complexity:

(i) The flat cosmological model, the simplest one-parameter fit to the combined SN + BAO + CMB data, appears on the top row of Table 3. In this model, cosmic acceleration started at $z_{\mathrm{acc}} = 0.76 \pm 0.010, 6.7 \pm 0.4\,\mathrm{Gyr}$ ago, but the cosmological constant began dominating over ordinary CDM only later at $z_{\mathrm{eq}} = 0.40 \pm 0.04, 4.3\,\mathrm{Gyr}$ ago.[45] (All confidence limits are 95%.). This static dark energy model, with $w_{\mathrm{DE}} = -1$, serves as a standard for comparison with the following 11 dynamical models.

(ii) The next three models are spatially curved cosmological constant, spatially flat constants w_{DE}, and flat pure generalized Chaplygin gas

$$\begin{gathered} P = -\frac{A}{\rho_{\mathrm{DE}}^{\alpha}}, \\ \left(\frac{H(z)}{H_0}\right)^2 = [A + (1-A)(1+z)^{3(1+\alpha)}]^{1/(1+\alpha)}, \\ A^{1/(1+\alpha)} = \Omega_{\Lambda 0}. \end{gathered} \tag{14}$$

These introduce a second parameter without any significant loss in goodness of fit (GoF). The broad uncertainties in the second parameter show the insignificance of going beyond the simple one-parameter flat cosmological constant model. In the constant w_{DE} model, cosmic acceleration started at $z_{\mathrm{acc}} = 0.81 \pm 0.30, 6.8 \pm 1.4\,\mathrm{Gyr}$ ago and the cosmological constant began dominating over ordinary CDM at $z_{\mathrm{eq}} = 0.44 \pm 0.20, 4.5 \pm 1.0\,\mathrm{Gyr}$ ago, slightly earlier than in ΛCDM.[45]

(iii) The next four models listed are the variable $w_{\mathrm{DE}}(z)$, spatially curved constant w_{DE}, generalized Chaplygin gas:

$$\begin{aligned} P &= -\frac{A}{\rho_{\mathrm{DE}}^{\alpha}}, \\ \left(\frac{H(z)}{H_0}\right)^2 &= \Omega_{K0}(1+z)^3 \\ &\quad + (1-\Omega_{K0})[A + (1-A)(1+z)^{3(1+\alpha)}]^{1/(1+\alpha)}, \end{aligned} \tag{15}$$

and flat, matter-dominated modified Cardassian polytropic,[46] which expands according to the modified Friedmann equation:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m0}(1+z)^3[1 + (\Omega_{m0}^{-q} - 1)(1+z)^{3q(n-1)}]^{1/q}. \tag{16}$$

These four models introduce a third parameter, with insignificant loss in GoF, but at the price of still more complexity.

(iv) The last four models listed are the spatially flat or curved ordinary Chaplygin gas and DGP models in Eq. (17) discussed in Sec. 6 and Appendix C. The flat models depend on only one parameter, $\Omega_{m0} \approx 0.27$, for which, in the

DGP case, $\beta \approx 1.4,\ r_c \sim 5.7\,\text{Gpc}$. The spatially curved DGP model requires $\Omega_{m0} = 0.27 \pm 0.03, \Omega_{K0} = 0.13 \pm 0.05$. These original DGP models cannot simultaneously fit the SN +BAO and CMB data[33] and have poor GoF ≈ 20%. The two ordinary Chaplygin models simply do not fit all the data. (Ignoring the BAO and CMB data, Szydlowski *et al.*[13] reached the opposite conclusion.) All four DGP and ordinary Chaplygin gas models show too rapid variation of $H(z)$ and they are rejected by their poor GoF.

All acceptable models agree at low redshift, exclude fast evolution such as would be predicted by many dark gravity models, and are now static or quasi-static. For example, the DGP models in Fig. 2, already at $z = 2$ show slopes $d_{\mathrm{M}}/dz = H(z)^{-1}$ about 10% faster than those obtained from the static classical cosmological constant model. The simplest and best fit to the combined data is the flat cosmological constant model on the first line of Table 3. The remaining seven acceptable quasi-static models on lines 2–8 of Table 3, with essentially the same GoF as ΛCDM, have their additional parameters so poorly constrained so that, in the worst cases, Davis *et al.*[3] did not quote their values. These complex models cannot be refuted by more high redshift supernovae, but only by much more weak lensing observations.[10] Although no present evidence requires dynamical gravitational dark energy, so long as these seven models are observationally allowed, we will go on to study the possible dynamical manifestations of gravitational vacuum energy.

5. Growth of Fluctuations can Distinguish Cosmodynamics

The kinematic degeneracy between dark energy and dark gravity will be lifted by the fluctuation growth function, provided the dark energy supports only isotropic stresses. In a mixture of cosmological fluids or with dynamical scalar fields, the equation of state is generally nonadiabatic: fluctuations propagate with an effective sound speed $c_s \neq c_a^2$. Since the entropic pressure fluctuations are proportional to $(1 + w)(c_s^2 - c_a^2)$, in the quasi-static limit $w(z) \sim -1$, they are small and insensitive to the effective sound speed. This minimizes the differences between the static and dynamic dark energy fluctuation growth factors that we would like to observe.

The integrated effects of the growth of fluctuations appear in:

Sunyaev–Zeldovich effects: Integrated energy effects appear dynamically in the thermal Sunyaev–Zeldovich effect (tSZ); integrated momentum effects appear kinematically in the kinematic Sunyaev–Zeldovich effect (kSZ), but are only second-order effects.

Sachs–Wolfe effects (ISW): Due to the evolution of the intervening gravitational potential on radiation (early Sachs–Wolfe effect), on vacuum energy (late Sachs–Wolfe effect). The nonlinear growth is the Rees–Sciama effect.

The fluctuation growth function will be best studied in the weak lensing convergence of light from galaxies at $0 < z < 5$, from neutral hydrogen at $6 < z < 20$, and ultimately from the CMB last scattering surface at $z = 1089$. Galaxy clustering also measures $g(z)$, but will require large corrections for the still uncertain baryonic composition and foreground noise. Projected observations may potentially distinguish static from dynamic "dark energy", but distinguishing dynamic dark energy from dark gravity will require a weak lensing shear survey, more ambitious than any now projected.

The original pure Chaplygin gas,[47–50] with equation of state $P(\rho) = -A/\rho$, energy density $\rho = \sqrt{A + B/a^6}$, and adiabatic sound speed $c_a^2 = -P/\rho = -w(z)$, is a unified dark matter/dark energy model, evolving from CDM at early times towards ΛCDM at late times. This model illustrates how, for the same macroscopic hydrodynamics, different microscopic field dynamics can lead to different effective sound speeds and growth of fluctuations. The hydrodynamic equation of state can be derived from either a canonical or noncanonical scalar field theory, for which the effective sound speed $c_s = 1$ or $\neq 1$, respectively, so that the fluctuation growth factors are different. If derived from the noncanonical purely kinetic Born–Infeld Lagrangian $\mathcal{L}(\mathcal{X}) = -V_0\sqrt{1 + \varkappa^2 X} = P$, $\rho = V_0/\sqrt{1 - \varkappa^2 X}$, $V_0^2 \equiv A$, the fluctuations remain adiabatic, and the effective sound speed $c_s^2 = c_a^2 = -w(z)$. But, if derived from the canonical scalar field with potential $V(\phi) = (\sqrt{A}/2)[\cosh 3\phi + 1/\cosh 3\phi]$,[50] entropic fluctuations make the effective sound speed $c_s^2 = 1 \neq c_a^2 = -w(z)$. (As mentioned in Sec. 4.3, this original pure 7 Chaplygin gas does not fit into the observed expansion history, but can be generalized to $P = -A/\rho^\alpha$, $c_a^2 = -\alpha w$, $\alpha = 0.06 \pm 0.10$, for which the effective sound speed $c_s^2 = -\alpha w \approx 0$. This makes the generalized Chaplygin gas indistinguishable from ΛCDM, which is the best fit to the expansion history.[47,50–52])

6. Modified Gravity: Dvali–Gabadadze–Porrati Brane Cosmology

Because dark energy is contrived, it requires fine tuning and cannot be directly detected in the laboratory or solar system. We now turn to dark gravity as an alternative, dynamical source of cosmological acceleration. These dark gravity alternatives arise naturally in braneworld theories, naturally incorporate a small space–time intrinsic curvature, and may try to unify "dark energy" and dark matter and early and late inflation. While fitted to the observed cosmological acceleration, they may also ultimately be tested in the solar system, galaxy or galaxy clusters.[5–8,31]

General relativity is a rigid metric structure incorporating general covariance (coordinate reparametrization invariance), the equivalence principle, and the local validity of Newtonian gravity with constant $G_{\rm N}$, in the weak field and nonrelativistic limits. General covariance implies four Bianchi identities on the Ricci curvature tensor. The linearity of the Einstein–Hilbert action in the Ricci scalar curvature makes the Einstein field equations second order, the two tensorial (graviton) degrees of freedom dynamic, and constrains the scalar and the vector $g_{\mu\nu}$ degrees to be nonpropagating.

General relativity differs from Newtonian cosmology only by the pressure or relativistic velocity effects, which are tested in the solar system and in cosmology (gravitational lensing of light, nucleosynthesis, dynamical age, large angular scale CMB, and late-time mass power spectrum). Therefore, modifications of general relativity must be sought, in order of scale, in laboratory violations of the equivalence principle (Eötvos experiments); in solar system tests (lunar ranging, deflection of light, and anomalous orbital precessions of the planets and Moon,[5,8,31,53,54] or a secular increase in the astronomical unit[6]); in galaxy and galaxy cluster number counts[7,55]; in gravitational weak lensing; in cosmological variation of Newton's G_{N} and other "constants"; in the enhanced suppression of fluctuation growth on large scales or at late times.

Because in general relativity only the metric's tensor degrees of freedom are propagating, modifying the Lagrangian R introduces additional scalar and vector degrees of freedom, represented by scalar or vector gravitational fields. The basic distinction between high- and low-curvature modifications of general relativity depends on the space–time curvature of their vacua. While high-curvature (ultraviolet) modifications have always been motivated by quantum gravity, low-curvature (infrared) modifications are now motivated by the discovery of the recently accelerating universe and can apparently be classical in origin.

6.1. *Four-dimensional modifications of general relativity*

For historical and didactic reasons, we begin by summarizing 4-dimensional metrical deformations of general relativity, which often appear as the projections of higher-dimensional theories and unnaturally incorporate Λ by fine tuning. In curved space–time, an S-matrix does not exist and causal paradoxes often appear.[14]

Scalar-tensor gravity is the oldest and the simplest extension of general relativity[56,57]: In the original Jordon Lagrangian, a scalar gravitational field, proportional to time-varying $1/G_{\mathrm{N}}$, couples linearly to the Ricci scalar R. After a conformal transformation to the Einstein frame, the scalar gravitational field is nonminimally coupled to matter, so that test particles do not move along geodesics of the Einstein metric. Instead, test particles move along the geodesics of the Jordon metric, so that the weak equivalence principle holds.[58]

Scalar-tensor theories modify Einstein gravity at all scales and must be fine-tuned to satisfy observational constraints. Nucleosynthesis and solar system constraints severely restrict any scalar field component, so that any dark gravity effects on the CMB or cosmological evolution must be very small.[59–61]

In higher-order $f(R)$ theories, the Lagrangian is no longer simply linear in the Ricci scalar R, so the equations of motion become fourth order, equivalent to scalar-tensor theories. $f(R)$ theories are liable to instability if either the kinetic or potential energies are negative. If the Lagrangian is a nonlinear function of only R, the kinetic energies are positive. But negative kinetic energies are unavoidable if the Lagrangian depends upon higher-order curvature invariants, such as $P \equiv R_{\mu\nu}R^{\mu\nu}$ or $Q \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$.[62,63] This same kinetic instability afflicts $f(R)$ Lagrangians involving derivatives of any curvature scalar, except total derivatives such as the Gauss–Bonnet invariant, which can be eliminated by partial integration.

The simplest low-curvature modification, replacing Einstein–Lagrangian density R by $R - \mu^4/R$,[64–66] leads to an accelerated expansion at low curvature $R \leq \mu^2 \sim H_0^2$, but is unstable because of the minus sign before $-\mu^4/R$. Outside matter, this theory is weakly tachyonically unstable and phenomenologically unacceptable.[67] Inside matter, this tachyonic instability is vastly and unacceptably amplified.[68] These tachyonic instabilities are, however, not generic and we can hope to avoid them by altering the dependence upon R.[69–71]

All $f(R)$ theories are conformally equivalent to scalar-tensor theories with vanishing Brans–Dicke parameter $\omega_{\mathrm{BD}} = 0$,[72–74] in which the scalar component, like the tensor component, couples minimally to matter in the Jordon frame, but not in the Einstein frame, dissociating space–time curvature from local matter density. Curvature constraints may then be satisfied in both the solar system (high density) and at high redshifts (low density).[70] This "chameleon mechanism"[75] does not work for the simple inverse-curvature theory, but must be applicable in any $f(R)$ theory that simultaneously satisfies the solar system and cosmological constraints. In practice, any acceptable $f(R)$ theory must mimic ΛCDM throughout cosmic history and must be fine-tuned to obtain the observed "dark energy" phase.[71]

All such theories can be fined-tuned to avoid potential instabilities and to satisfy supernova and solar system constraints,[63,67,76,77] but not cosmological constraints.[78]

TeVeS (relativistic MOND theory): With the addition of an additional vector gravitational field, we could explain flat galactic rotation curves and the Tully–Fisher relation without invoking dark matter, and could possibly unify dark matter and "dark energy".[79] Because gravitons and matter have different metric couplings, TeVeS predicts that gravitons should travel on geodesics different from photon and neutrino geodesics, with hugely different arrival times from supernova pulses. It also predicts an insufficient power in the third CMB acoustic peak.[80] In any case, now that colliding galaxies and WMAP3 data require dark matter,[15] the motivation for TeVeS disappears.

6.2. *Extra-dimensional (braneworld) modifications*

In extra-dimensional braneworld theories inspired by string theory,[81,82] scalar fields appear naturally as dilatons and modify Einstein gravity at high curvature by brane warping,[23,25] or at low curvature by brane leakage of gravity.[9] These theories calculate Λ by fine tuning of the compactified dimensions. Some of these dark gravity theories potentially unify both early and late inflation, but only by including some dark energy components.

When quantized, these theories encounter serious theoretical problems (ghosts, instabilities, and strong coupling problems) and they are not now derivable from fundamental quantum field theories. Until these problems can be overcome, these theories must be regarded as effective field theories, incorporating an extremely low infrared scale at low space–time curvature, unlike other effective field theories, which incorporate ultraviolet parameters.

In the original DGP model,[9,83,84] leakage of gravity into the 5-dimensional bulk leads to a Friedmann equation,

$$H^2 + \frac{k}{a^2} - \frac{H}{r_c} = \frac{\varkappa^2 \rho}{3}, \tag{17}$$

modified on the 4-dimensional brane by the additional curvature term H/r_c at the cosmological scale r_c. In our matter-dominated epoch, this modified Friedmann equation is

$$\begin{gathered} \left(\frac{H(z)}{H_0}\right)^2 = \left[\frac{1}{2\beta} + \sqrt{\left(\frac{1}{2\beta}\right)^2 + \Omega_{m0}(1+z)^3}\,\right]^2 + \Omega_{K0}(1+z)^2, \\ 1 \equiv \Omega_{m0} + \Omega_{K0} + \frac{\sqrt{1-\Omega_{K0}}}{\beta}, \end{gathered} \tag{18}$$

where the terms inverse in $\beta := H_0 r_c$ express the weakening of gravity on the brane at large scales $a > r_c$, due to leakage into the 5-dimensional bulk. This modified Friedmann equation interpolates between the past matter-dominated universe for small scales $a \ll \beta^{2/3}$, and the future de Sitter universe with constant Hubble expansion $H_{\mathrm{dS}} := 1/r_c$ for scales $a \gg 1$.

For the intermediate value $\beta = 1.39$, $r_c \sim 5.7\,\mathrm{Mpc}$, the universe began its late acceleration at $z_{\mathrm{acc}} = (2\Omega_{m0}/\beta^2)^{1/3} - 1 \sim 0.58$, as shown in Fig. 1. This is the original DGP model on the tenth line of Table 3, which turns out to be indistinguishable from the flat DGP model on the ninth line.

About any isolated spherically symmetric condensation of Schwarzschild radius $r_{\mathrm{S}} := 2G_{\mathrm{N}}M/c^2$, the self-accelerating metric

$$g_{\mathrm{tt}} = \frac{1 - r_{\mathrm{S}}}{2r} + \sqrt{\frac{r_{\mathrm{S}}^2 r}{2r_\star^3}}, \quad g_{\mathrm{rr}}^{-1} = \frac{1 + r_{\mathrm{S}}}{2r} - \sqrt{\frac{r_{\mathrm{S}}^2 r}{8r_\star^3}}, \quad r \lesssim r_\star, \tag{19}$$

so that Einstein gravity obtains only up to Vainshtein scale[10]:

$$r_\star := (r_{\mathrm{S}} r_c^2)^{1/3} \sim (H_0 r_{\mathrm{S}})^{1/3} H_0^{-1} \ll H_0^{-1}. \tag{20}$$

This intermediate scale, $r_S \ll r_\star \ll H_0^{-1}$, is also where the growth of fluctuations would change from Einstein gravity over to the linearized DGP or to scalar-tensor Brans–Dicke gravity, with an effective Newton's constant slowly decreasing by no more than a factor of two.[28] For cosmological scale $r_c \to \infty$, the DGP-modified Friedmann equation reduces to the Einstein–Friedmann equation, but the DGP metric (C3) still does not reduce to Schwarzschild metric: there are neither iron sphere nor Birkhoff's theorems in the DGP geometry.

The original flat DGP model can be generalized[85] to

$$H^2 - H^{\alpha} r_c^{\alpha-2} = \frac{\varkappa^2 \rho}{3}, \quad 1 = \Omega_{m0} + \beta^{\alpha-2}, \tag{21}$$

which is equivalent to a "dark energy" $\rho_{\mathrm{DE}} := 3M_{\mathrm{P}}^2 H^2 - \rho = 3M_{\mathrm{P}}^2 H^{\alpha} r_c^{\alpha-2}$, $w_{\mathrm{DE}} = -1 + \alpha/2$. This generalization reduces to the original flat DGP form for $\alpha = 1$, but otherwise interpolates between the Einstein–de Sitter model for $\alpha = 2,\ \beta = \infty$ and the flat classical cosmological constant model for $\alpha = 0$. For small α, it describes a slowly varying cosmological constant. In the self-accelerating solution of the original DGP model,[9,83,84] discussed in Appendix C, gravity leaks into the 5-dimensional bulk at cosmological scales greater than r_c, weakening gravity on the 4-dimensional brane. This leads to a 4-dimensional Friedmann equation:

$$H^2 + \frac{k}{a^2} - \frac{H}{r_c} = \frac{\varkappa^2 \rho}{3}. \tag{22}$$

Defining $\beta := r_c H_0$, which can be written as

$$\begin{gathered} \left(\frac{H(z)}{H_0}\right)^2 = \left[\frac{1}{2\beta} + \sqrt{\left(\frac{1}{2\beta}\right)^2 + \Omega_{m0}(1+z)^3}\,\right]^2 + \Omega_{K0}(1+z)^2, \\ 1 \equiv \Omega_{m0} + \Omega_{K0} + \frac{\sqrt{1-\Omega_{K0}}}{\beta}, \end{gathered} \tag{23}$$

interpolating between a past matter-dominated universe and a future de Sitter universe.

Figure 3 shows the growth factors in this DGP dark gravity and its dark energy mimic both evolving substantially faster than in the cosmological constant model ΛCDM. This figure also shows a relatively smaller difference between dark gravity DGP and its dark energy mimic. In the next decade, weak lensing observations may distinguish between static and dynamic "dark energy", but not between dark gravity and dark energy.

As we have discussed in Sec. 4.3, the original DGP models cannot simultaneously fit the SN + BAO and the CMB data. We have used these models only to show the importance of the 5D Weyl tensor in any braneworld dynamics and to illustrate how dynamics is better tested in the growth of fluctuations (Fig. 3) than in the homogeneous expansion history (Fig. 2). In any realistic model, because the evolution is, at most, quasi-static, any dynamical effect on the growth of fluctuations will be minimal, and will be best studied in the weak lensing convergence of light from the galaxies at $0 < z < 5$, from neutral hydrogen at $6 < z < 20$, and ultimately from the CMB last scattering surface at $z = 1089$.[4,11] Galaxy clustering also measures $g(z)$, but requires large corrections for baryonic composition and foreground noise to reduce their large systematic errors.

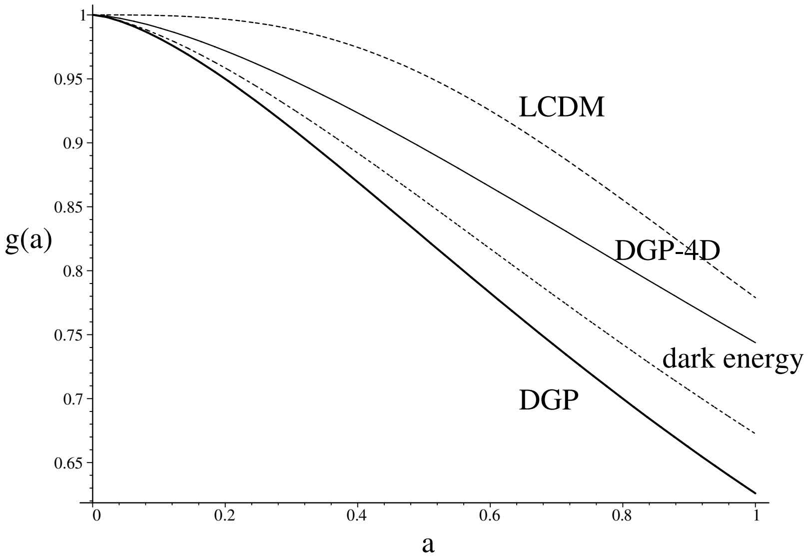

Fig. 3. The linear growth history $g(a) := \delta/a$ for flat ΛCDM (long-dashed), DGP dark gravity (thick solid) and dark energy (short-dashed) models in Fig. 2. DGP-4D (thin solid) shows the incorrect result that would be obtained by neglecting perturbations of the DGP-5D Weyl tensor. The 5D Weyl tensor perturbations thus distinguish the static dark energy model from the two dynamical dark gravity and mimicking dark energy models. In the DGP dark gravity model, Newton's "constant" weakens with time, so that its growth suppression evolves even faster than in its mimicking dark energy model. Figure from Ref. 17.

7. Cosmological Constant — Fine-Tuned Dark Energy, or Modified Gravity?

7.1. *Phenomenological conclusions: Vacuum energy is now static or quasi-static*

We have reviewed the present and prospective observations of "dark energy" in order to emphasize the differences between kinematical and dynamical observations, between static and dynamic "dark energy", and between dark energy and dark gravity. We conclude:

- Cosmological acceleration is explicable by either a small fine-tuned cosmological constant or by "dark energy" that is now nearly static. If dynamic, this "dark energy" is either an additional, ultralight negative pressure material within general relativity, or a low-curvature modification of Einstein's field equations.
- The simplest and best fit to the expansion history, the classical cosmological constant model, interprets "dark energy" as a classical intrinsic space–time curvature, giving geometric structure to empty space. This classical interpretation distinguishes "gravitational vacuum energy" from the ground state of quantum matter, and renounces any attempt to explain its small value as a quantum vacuum energy.

- The observed homogeneous expansion history may also be fitted by "dark energy" decaying from its huge primordial value and now nearly static. This static or quasi-static "dark energy" presently observed, whether dark energy or dark gravity requires fine tuning.
- The inhomogeneity growth rate potentially distinguishes between static and dynamic "dark energy" and between dark energy and dark gravity. Because the "dark energy" is now static or nearly static, differences in the large-scale angular power spectrum, mass power spectrum, or gravitational weak lensing[86] will be small, but may distinguish static from dynamic "dark energy". Distinguishing between dark energy and dark gravity will remain more problematic.
- No model yet explains the cosmological constant problem, why quantum vacuum energies apparently do not gravitate. Nor does any model explain the cosmic coincidence, why we observers live at a time when the matter and vacuum energy densities are comparable.

Low-curvature modifications of Einstein gravity are conceptually less contrived than fine-tuned dark energy, explain the cosmological acceleration as a natural consequence of geometry, and may unify early and late inflation. Geometric modifications may be intrinsic in four dimensions, or may arise naturally in braneworld theories. Invoked in the first place to explain recent cosmological acceleration, these low-curvature modifications of Einstein gravity may even be testable by refined solar system or galaxy observations. The outstanding problem in both dark energy and dark gravity remains the significance of the cosmic coincidence, which unless fine-tuned, clearly refers to the role of conscious observers.

7.2. *Metaphysical conclusions: The role of observers*

The expanding scope of physics has always required new principles, such as the relativity, equivalence, complementarity, and uncertainty principles, which limit what can be observed. But cosmology differs from other simply descriptive physical sciences in two ways: first, by cosmic variance: observations are confined to our past light cone and to our finite Hubble volume; second, by evolutionary character: cosmogeny depends on the initial conditions, as well as on the physical laws. These two characteristics may call for a new paradigm, selecting among the possible universes or, at least, among different cosmological constants.[87]

The observed cosmological acceleration requires an unnaturally tiny gravitational vacuum energy, inequivalent to the material vacuum energy (cosmological constant problem). Why are we living and observing now, when the material vacuum energy density has reduced to nearly around the observed gravitational vacuum energy density (cosmological coincidence problem)? The gravitational vacuum energy was undetectable when the universe was much younger. In the distant future, when the universe becomes nearly de Sitter and nearly static, our observations will be confined to the bound local group, so that the cosmological expansion itself will become undetectable.[21,22]

It should come as no surprise if the cosmic coincidence problem calls for a new ontological principle, limiting what can be observed about the gravitational vacuum. The mind is the window to reality, but we are now only beginning to understand how the mind perceives reality. Cognition, consciousness, and feeling are only now becoming physical observables, but it is already clear that intelligence depends on our evolved brain structure and on past experience, and allows many different interpretations of reality. This ensemble of conceivable theories for our universe[88] may or may not be realized as subuniverses of a megauniverse (landscape), or may recur periodically in a bouncing universe, or may evolve from one universe to another by natural selection.[89] In any case, if the cosmic coincidence cannot be explained by a fundamental theory, some form of anthropic selection (weak anthropic principle) is required. We would emphasize that the present data suggests a particularly restrictive form of anthropic selection, that *we cannot distinguish between dark energy and dark gravity.* This restrictive anthropic principle selects static "dark energy" or ΛCDM.

In the absence of critically distinguishing observations, any theory must be judged by its usefulness and economy. Perhaps all aspects of reality, including observers' cognition, will ultimately be reducible to deterministic physical principles. But until then, some physicists find it useful to invoke a new cosmological principle, such as the anthropic principle in a single universe or landscapes in a multiuniverse. Choosing between such a new cosmological principle and a strict reductionism is today a subjective choice between still-hopeful string theorists and more-doubtful phenomenologists.[90]

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References

1. J. A. Gu and W. Y. Hwang, *Phys. Rev. D* **65** (2001) 024003.
2. V. Sahni and A. A. Starobinsky, *Int. J. Mod. Phys. D* **15** (2006) 2105.
3. T. M. Davis *et al.* (2007), arXiv:astro-ph/0701510v1, Table 2, Fig. 7.
4. M. Ishak, A. Upadhye and D. Spergel, *Phys. Rev. D* **74** (2006) 043513 [arXiv:astro-ph/0507184].
5. A. Lue and G. Starkman, *Phys. Rev. D* **67** 064002 (2003) [arXiv:astro-ph/0212083].
6. L. Iorio (2005), arXiv:gr-qc/0510059.
7. L. Iorio, *J. Cosmol. Astropart. Phys.* **9** (2005) 6.
8. A. Lue, *Phys. Rep.* **423** (2006) 1.

9. G. Dvali, G. Gabadadze and M. Porrati, *Phys. Lett. B* **484** (2000) 112, [DGP].
10. A. Riess *et al.*, *Astrophys. J* **659** (2007) 98.
11. A. Albrecht *et al.*, (2006), [DETF], arXiv:astro-ph/0609591.
12. T. Padmanabhan, *Curr. Sci.* **88** (2005) 1057.
13. M. Szydlowski, A. Kurek and A. Krawiec, *Phys. Lett. B* **642** (2006) 206 [arXiv:astro-ph/0604327v2].
14. E. Babichev, V. Mukhanov and A. Vikman (2007), arXiv:0708.0561 [hep-th].
15. D. Spergel *et al.*, (2006), arXiv:astro-ph/0603449, Tables 2, 9 [WMAP3].
16. E. Rozo *et al.*, *Astrophys. J.* (2007), arXiv:astro-ph/0703571.
17. K. Koyama and R. Maartens, *J. Cosmol. Astropart. Phys.* **0601** (2006) 016 [arXiv:astro-ph/0511634].
18. W. Buchmuller and N. Dragon, *Phys. Lett. B* **207** (1988) 292.
19. W. Unruh, *Phys. Rev. D* **40** (1989) 1048.
20. S. K. Lamoreaux, *Phys. Today* **60** (2007) 40.
21. T. Rothman and G. Ellis, *Observatory* **107** (1987) 24.
22. L. Krauss and R. Scherrer, *Gen. Relativ. Gravit.* **39** (2007) 1545.
23. L. Randall and R. Sundrum, *Phys. Lett. B* **83** (1999) 3370.
24. N. Arkani-Hamad, S. Dimopoulos and G. Dvali, *Phys. Lett. B* **429** (1998) 263.
25. P. Binutray, C. Deffayet and D. Langlois, *Nucl. Phys. B* **565** (2000) 269.
26. E. Milne, *Quart. J. Math.* (Oxford) **5** (1934) 64.
27. W. McCrea and E. Milne, *Quart. J. Math.* (Oxford) **5** (1934) 73.
28. A. Lue, R. Scoccimaro and G. Starkman, *Phys. Rev. D* **69** (2004) 124015.
29. M. Ishak *et al.*, (2007), arXiv:0710.4726v1 [astro-ph].
30. G. Dvali, A. Gruzinov and M. Zaldarriaga, *Phys. Rev. D* **68** (2003) 024012.
31. M. Sereno and P. Jetzer, *Mon. Not. R. Astron. Soc.* **371** (2006) 626 [arXiv:astro-ph/0606197].
32. Y. Wang and P. Mukherjee, *Astrophys. J* **650** (2006) 1.
33. M. Fairbairn and A. Goobar, *Phys. Lett. B* **642** (2006) 432.
34. D. Eisenstein *et al.*, *Astrophys. J.* **633** (2005) 560 [SDSS Luminous Red Galaxy Survey].
35. S. Perlmutter *et al.*, *Astrophys. J.* **517** (1999) 565 [Supernova Cosmology Project].
36. P. Astier *et al.*, *Astron. Astrophys.* **447** (2006) 31 [Supernova Legacy Survey SNLS].
37. G. Miknaitis *et al.*, *Astrophys. J.* (2007), arXiv:astro-ph/0701043.
38. Y. Wang and M. Tegmark, *Phys. Rev. Lett.* **92** (2004) 241302.
39. L. Maor, R. Brustein and P. Steinhardt, *Phys. Rev. Lett.* (2001).
40. E. Linder and D. Huterer, *Phys. Rev. D* **72** (2005) 043509.
41. R. R. Caldwell and E. V. Linder, *Phys. Rev. Lett.* **95** (2005) 141301.
42. M. Chevalier and D. Polarski, *Int. J. Mod. Phys. D* **10** (2001) 213 [arXiv:gr-qc/0009008].
43. E. Linder, *Phys. Rev. Lett.* **90** (2003) 091301.
44. Y. Wang and K. Freese, *Phys. Lett. B* **632** (2006) 449.
45. A. Melchiorri, L. Pagano and S. Pandolfi (2007), arXiv:0706.1314 [astro-ph].
46. P. Gondolo and K. Freese, *Phys. Rev. D* **68** (2003) 063509.
47. V. Gorini *et al.*, *J. Cosmol. Astropart. Phys.* (2008), arXiv:0711.4242.
48. U. Alam and V. Sahni, *Phys. Rev. D* **73** (2006) 084024.
49. R. Maarten and E. Majerotto, *Phys. Rev. D* **74** (2006) 023004.
50. M. C. Bento *et al.*, *Phys. Rev. D* (2005), arXiv:astro-ph/0412638.
51. L. Amendola *et al.*, *J. Cosmol. Astropart. Phys.* **309** (2003) 5 [arXiv:astro-ph/0304325].
52. Z. H. Zhu, *Astron. Astrophys.* (2004).

53. T. Damour, *Phys. Lett. B* **592** (2004) 186.
54. G. Gabadadze and A. Iglesias, *Phys. Lett. B* **632** (2006) 617.
55. M. Sereno and P. Jetzer, *Phys. Rev. D* **73** (2006) 063004 [arXiv:astro-ph/0602438].
56. Y. Fujii and K.-I. Maeda, *The Scalar-Tensor Theory of Gravitation* (Cambridge University Press, Cambridge, England, 2003).
57. S. Capozziello, V. F. Cardone and A. Troisi, *Phys. Rev. D* **71** (2005) 043503.
58. B. Boisseau *et al.*, *Phys. Rev. Lett.* **85** (2000) 2236 [arXiv:gr-qc/0001066].
59. B. Bertotti, L. Das and P. Tortora, *Nature* **425** (2003) 374.
60. S. Capozziello, V. P. Cardone and A. Troisi, *J. Cosmol. Astropart. Phys.* **0608** (2006) 001 [arXiv:astro-ph/0602349].
61. R. Catena *et al.* (2004), arXiv:astro-ph/0406152.
62. M. Ostrogradski, *Mem. Ac. St. Petersbourg VI* 4 (1850) 385.
63. R. Woodard (2006), arXiv:astro-ph/0601672.
64. S. M. Carroll *et al.*, *Phys. Rev. D* **70** (2004) 043528.
65. S. M. Carroll *et al.*, *Phys. Rev. D* **71** (2005) 063513 [arXiv:astro-ph/0410031].
66. S. Nojiri and S. Odintsov, *Int. J. Geom. Methods Mod. Phys.* **4** (2007) 115 [arXiv:hep-th/0601213].
67. M. Soussa and R. Woodard, *Gen. Relativ. Gravit.* **36** (2004) 855.
68. A. D. Dolgov and M. Kawasaki, *Phys. Lett. B* **573** (2003) 1.
69. A. Starobinsky, *J. Exp. Theor. Phys. Lett.* **86** (2007) 157 [arXiv:0706.2041].
70. W. Hu and I. Sawicki, *Phys. Rev. D* (2008), arXiv:0705.3199.
71. S. Appleby and R. Battye (2007), arXiv:0705.3199v2.
72. P. Teyssandier and P. Tourrenc, *J. Math. Phys.* (1983).
73. G. Olmo, *Phys. Rev. D* **72** (2005) 083505.
74. S. Capozziello, S. Nojiri and S. D. Odintsov, *Phys. Lett. B* **634** (2006) 93 [arXiv:hep-th/0512118].
75. L. Faulkner *et al.* (2006), arXiv:astro-ph/0612569.
76. S. Nojiri and S. Odintsov, *Gen. Relativ. Gravit.* **36** (2004) 1765.
77. S. Nojiri and S. Odintsov, *Phys. Rev. D* **68** (2003) 123512.
78. L. Amendola, D. Polarski and S. Tsujikawa, *Phys. Rev. Lett.* **98** (2007) 131302.
79. J. D. Bekenstein, *Phys. Rev. D* **70** (2004) 083509.
80. C. Skordis *et al.*, *Phys. Rev. Lett.* **96** (2006) 011301 [arXiv:astro-ph/0505519].
81. T. Damour and A. Polyakov, *Nucl. Phys. B* (1994).
82. T. Damour and A. Polyakov, *Gen. Relativ. Gravit.* (1994).
83. C. Deffayet, *Phys. Lett. B* **502** (2001) 199 [arXiv:hep-ph/0010186].
84. C. Deffayet *et al.*, *Phys. Rev. D* **66** (2002) 024019.
85. G. Dvali and M. Turner (2003), arXiv:hep-th/0602178.
86. H. Hockstra *et al.*, *Astrophys. J.* **647** (2006) 116 [Canada-France-Hawaii Telescope], arXiv:astro-ph/0511089.
87. G. Ellis, *Handbook of Philosophy of Physics* (Elsevier, 2006).
88. S. Bludman and M. Ross, *Phys. Rev. D* **65** (2002) 043503.
89. L. Smolin, *The Trouble with Physics* (Houghton Mifflin, 2006), ISBN 13: 978-0-618-55105-7.
90. S. Weinberg, *Living in the Multiverse* (Cambridge University Press, 2006).